MONTH 9

Counting & Combinatorics

Organize, Count & Generalize

Month 9 teaches students to count complicated possibilities without listing them randomly. Through visual models, organized lists, combinations, arrangements, paths, and schedules, students discover how structure makes difficult counting problems manageable.

WHY THIS MONTH MATTERS

Students move from solving individual problems toward organizing evidence, selecting representations, and explaining strategies that can be reused in unfamiliar situations.

Students Should Be Able To:

- Create organized lists and tables.
- Use multiplication to count independent choices.
- Recognize when order matters.
- Use diagrams to count paths and pairings.
- Check that possibilities are not repeated.
- Explain a counting strategy clearly.

Skills We Will Practice

- Organized counting
- Tree diagrams
- Fundamental counting principle
- Combinations
- Permutations
- Systematic casework

This Month's Activities

- Outfit Builder
- Grid-Path Challenge
- Handshake Circle
- Topping Teams
- Seat the Team
- Round-Robin Schedule

Connection to Month 12

The month develops a different kind of mathematical maturity: organizing possibilities, evaluating uncertainty, or interpreting evidence. These habits will be essential during the final multi-topic Math Mania Challenge.

ACTIVITY 1

Outfit Builder

Solve, represent, and explain your reasoning.

THE CHALLENGE

A student has 3 shirts, 2 pairs of pants, and 2 pairs of shoes. How many different outfits can be made?



WHAT TO RECORD

Record the total number of outfits.

Hint: Multiply the number of choices at each step.

FINISHED EARLY? TRY THESE CHALLENGES

1. Add 2 hat choices. How many complete outfits are now possible?

2. With 4 shirts instead of 3, how many outfits are possible without hats?

Show Your Strategy

FINAL ANSWER: _____

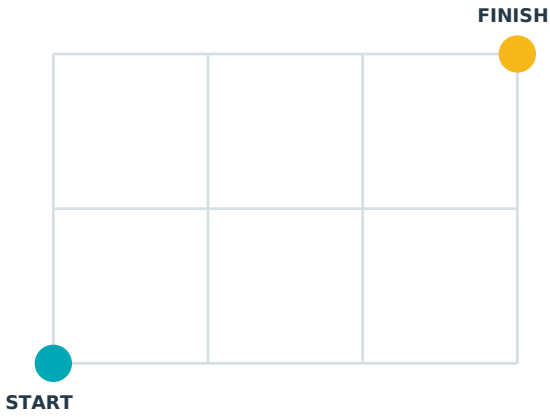
ACTIVITY 2

Grid-Path Challenge

Solve, represent, and explain your reasoning.

THE CHALLENGE

A robot must travel from START to FINISH using exactly 3 moves right and 2 moves up. How many shortest paths are possible?



WHAT TO RECORD

Count every different ordering of R,R,R,U,U.

Hint: Organize paths by the location of the two up-moves.

FINISHED EARLY? TRY THESE CHALLENGES

1. How many shortest paths use 4 right moves and 3 up moves?

2. How many original paths begin with an up move?

Show Your Strategy

FINAL ANSWER: _____

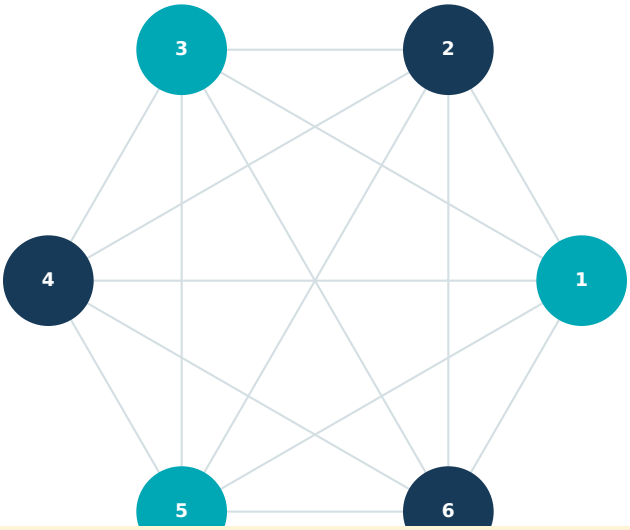
ACTIVITY 3

Handshake Circle

Solve, represent, and explain your reasoning.

THE CHALLENGE

Six students each shake hands with every other student exactly once. How many handshakes occur?



WHAT TO RECORD

Each pair should be counted only once.

Hint: Try 1+2+3+4+5.

FINISHED EARLY? TRY THESE CHALLENGES

1. How many handshakes occur among 8 students?

2. If one pair does not shake hands, how many handshakes remain?

Show Your Strategy

FINAL ANSWER: _____

ACTIVITY 4

Topping Teams

Solve, represent, and explain your reasoning.

THE CHALLENGE

Choose exactly 2 different toppings from 5 choices. How many two-topping combinations are possible?



P=pepper M=mushroom O=olive B=broccoli T=tomato

WHAT TO RECORD

Pepper + mushroom is the same as mushroom + pepper.

Hint: Make an organized list beginning with each topping.

FINISHED EARLY? TRY THESE CHALLENGES

1. How many combinations use exactly 3 of the 5 toppings?

2. If first and second toppings have different roles, how many ordered choices exist?

Show Your Strategy

FINAL ANSWER: _____

ACTIVITY 5

Seat the Team

Solve, represent, and explain your reasoning.

THE CHALLENGE

Four students will sit in four chairs in a row. How many different seating arrangements are possible?



WHAT TO RECORD

Here, changing the order creates a new arrangement.

Hint: There are 4 choices, then 3, then 2, then 1.

FINISHED EARLY? TRY THESE CHALLENGES

1. How many arrangements are possible for 5 students?

2. How many arrangements place Ava in the first chair?

Show Your Strategy

FINAL ANSWER: _____

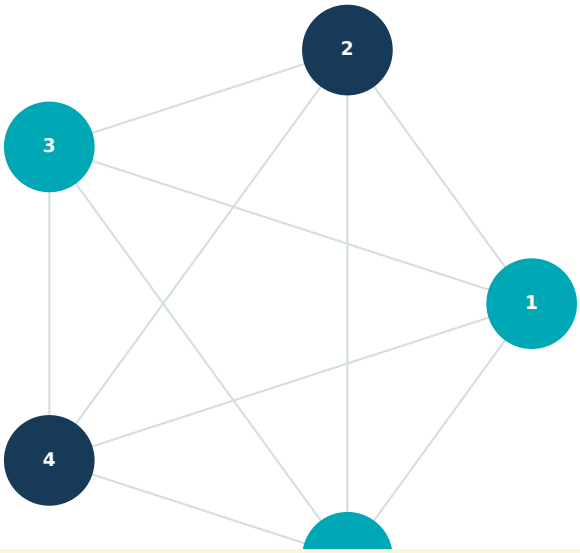
ACTIVITY 6

Round-Robin Schedule

Solve, represent, and explain your reasoning.

THE CHALLENGE

Five teams must each play every other team exactly once. How many games are needed?



WHAT TO RECORD

Represent each matchup with one connecting line.

Hint: This has the same structure as a handshake problem.

FINISHED EARLY? TRY THESE CHALLENGES

1. How many games are needed for 8 teams?

2. If 5 teams play each other twice, how many games are needed?

Show Your Strategy

FINAL ANSWER: _____

FINAL ACTIVITY

Festival Planner

Plan, represent, test, and communicate.

YOUR MISSION

Design a one-day math festival with 2 themes, 3 snack choices, and 2 featured games. Determine the number of possible festival plans. Then choose 2 student hosts from a group of 4 and determine the number of possible host pairs. Show both counting methods visually.

Project Checklist

- Write a clear question or goal.
- Choose an organized mathematical representation.
- Show calculations and label evidence.
- Explain the strategy or conclusion.
- Check the result and identify a limitation or possible revision.

Planning Space

INSTRUCTOR SECTION

Month 9 Answer Key

Counting & Combinatorics

1. Outfit Builder

3 x 2 x 2 = 12 outfits.

2. Grid-Path Challenge

Choose the positions of the two U moves among five moves: 10 paths.

3. Handshake Circle

5+4+3+2+1 = 15 handshakes.

4. Topping Teams

Choose 2 of 5: 10 combinations.

5. Seat the Team

4 x 3 x 2 x 1 = 24 arrangements.

6. Round-Robin Schedule

Each pair plays once: 10 games.

Project

2 x 3 x 2 = 12 festival plans; choose 2 of 4 = 6 host pairs.

INSTRUCTOR SECTION

Month 9 Extension Answers

Solutions for the finished-early questions

1. Outfit Builder

Challenge 1: 2 x 12 = 24 outfits.

Challenge 2: 4 x 2 x 2 = 16 outfits.

2. Grid-Path Challenge

Challenge 1: Choose 3 up-move positions among 7: 35 paths.

Challenge 2: Fix U first, then arrange R,R,R,U: 4 paths.

3. Handshake Circle

Challenge 1: 7+6+5+4+3+2+1 = 28.

Challenge 2: 28 - 1 = 27.

4. Topping Teams

Challenge 1: Choose 3 of 5: 10 combinations.

Challenge 2: 5 x 4 = 20 ordered choices.

5. Seat the Team

Challenge 1: 5 x 4 x 3 x 2 x 1 = 120.

Challenge 2: Fix Ava first; arrange the other four: 24.

6. Round-Robin Schedule

Challenge 1: Choose 2 of 8: 28 games.

Challenge 2: 10 pairings x 2 = 20 games.